

# Long tunneling contact as a probe of fractional quantum Hall neutral edge modes

B. J. Overbosch<sup>1</sup> and Claudio Chamon<sup>2</sup><sup>1</sup>*Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139, USA*<sup>2</sup>*Department of Physics, Boston University, Boston, Massachusetts 02215, USA*

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We study the tunneling current between edge states of quantum Hall liquids across a single long-contact region and predict a resonance at a bias voltage set by the scale of the edge velocity. For typical devices and edge velocities associated with charged modes, this resonance occurs outside the physically accessible bias domain. However, for edge states that are expected to support neutral modes, such as the  $\nu=\frac{2}{3}$  and  $\nu=\frac{5}{2}$  Pfaffian and anti-Pfaffian states, the neutral velocity can be orders of magnitude smaller than the charged mode and if so the resonance would be accessible. Therefore, such long tunneling contacts can resolve the presence of neutral edge modes in certain quantum Hall liquids.

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## I. INTRODUCTION

Quantum Hall (QH) states are incompressible quantum fluids where all bulk excitations are gapped, but gapless modes exist at the boundaries. In the integer effect, edge states can be understood in a simple way for noninteracting electrons,<sup>1</sup> with an edge channel matching each filled Landau level in the bulk, as the Landau bands bend at the edges of the system due to the confining potential and cross the Fermi level. In the fractional effect the situation is richer, and there is a one-to-one relation due to gauge invariance that ties the bulk states, classified by 2+1D Chern-Simons theories, and the gapless edge modes.<sup>2</sup> Depending on the bulk filling fraction or the details of edge confinement, the edge theory may contain neutral modes, in addition to a charge mode that carries the quantized Hall currents. For example, even for a  $\nu=1$  QH state, neutral modes are present if the edge is smooth or reconstructed.<sup>3</sup> For fractional QH states, even for sharply defined edges, neutral modes may be present. Such is the case for  $\nu=\frac{2}{3}$  states,<sup>4,5</sup> as well as for the  $\nu=\frac{5}{2}$  Pfaffian and anti-Pfaffian non-Abelian states,<sup>6,7</sup> and the situation becomes even richer if the edges of such states undergo reconstructions.<sup>8</sup>

Chiral charge modes, which cannot be localized by disorder, are closely tied to the quantization of the Hall conductance; hence the existence of these modes is unavoidable. Experiments have been designed to probe the propagation of these charge modes, in particular to measure their wave velocity.<sup>9,10</sup> On the other hand, to the best of our knowledge, there has not been any experimental result that confirms the existence of the neutral modes.

There are a number of reasons as to why one should seriously look into ways of detecting neutral edge modes. For example, there are theoretically unresolved experimental findings on tunneling on the edges of QH liquids in cleaved-edge overgrown samples<sup>11,12</sup> which could be better understood if information on the neutral modes were available. More specifically, in these experiments one measures a non-linear  $I$ - $V$  characteristic of Luttinger liquid behavior at the edges; however, the power-law exponent is not in agreement with theoretical predictions.<sup>2,13</sup> Instead, these exponents match those obtained if one had only the charge mode and no

neutral ones. However, one cannot construct an operator for creating an excitation with the charge of an electron and fermionic statistics using the charge mode alone. Hence, the neutral modes are both the champions and the villains lingering over the resolution of this puzzle, and this has led to proposals that the neutral modes may be either extremely slow<sup>14</sup> or topological and nonpropagating at all.<sup>15</sup> Another reason to probe neutral modes is that, in the case of the interesting non-Abelian states, these are the modes that carry the nonlocal information of the order and twinning of edge quasiparticles.

The objective of this paper is to propose a way to probe neutral edge modes. The proposed setup consists of a long-contact region, or quantum long contact (QLC), in which there are several interfering paths for tunneling charge from two opposite edges of a Hall bar, as depicted in Fig. 1, resembling an ac Josephson junction. The idea of exploring interference between tunneling paths is reminiscent of a two-point-contact interferometer<sup>16</sup> (2PC) for probing quasiparticle statistics. Both methods are sensitive to neutral edge modes, the main difference being the observation window: the long-contact setup probes slower edge velocities than the two-point-contact setup.

We find that coherent tunneling inside a QLC gives rise to a resonance in the tunneling current at zero temperature for a bias voltage  $V_{\text{res}}$  given by

$$\frac{eV_{\text{res}}}{\hbar} = \frac{vW}{\ell_B^2}, \quad (1)$$

where  $W$  is the width of the QLC,  $\ell_B$  is the magnetic length, and  $v$  is the slowest edge velocity associated with the tun-

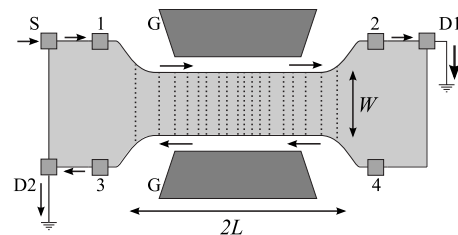


FIG. 1. Tunneling between two edges in a QLC does not occur at a single site but rather over a range of positions along the edge. Arrows indicate propagation direction of current.

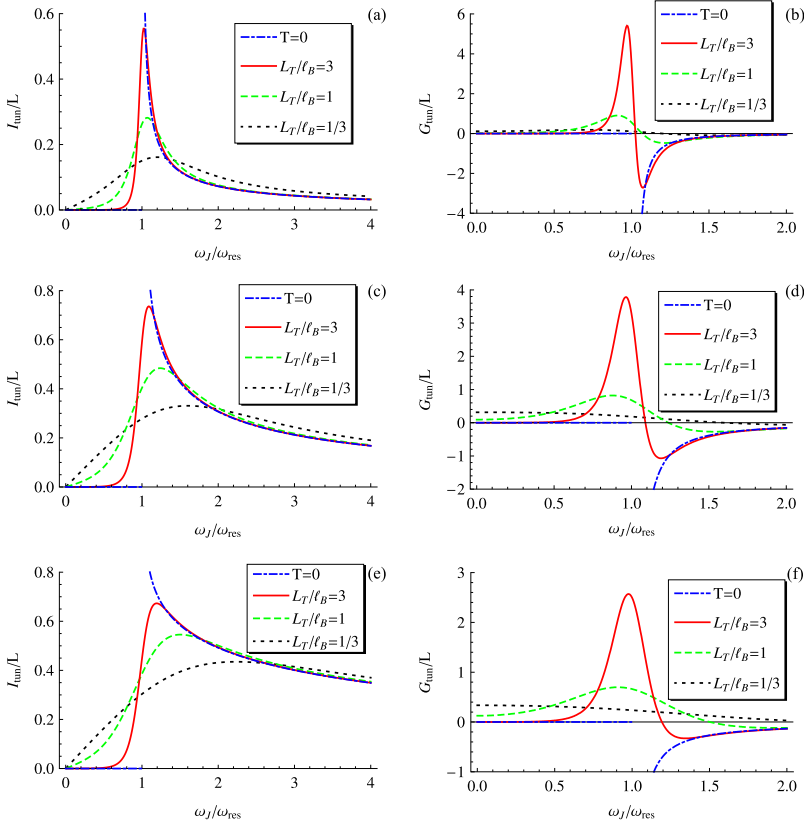


FIG. 2. (Color online) Plots of the tunneling current per unit length (left column) and differential tunneling conductance per unit length (right column) for three states: the Pfaffian [(a) and (b)], the anti-Pfaffian [(c) and (d)] and the  $\nu=2/3$  [(e) and (f)] state. The three states differ in their values for  $e^*$ ,  $g_c$ , and  $g_n$ . Plotted are  $I_{\text{tun}}/L$  and  $G_{\text{tun}}/L$  as function of bias voltage at zero and finite temperatures. At  $T=0$  the current and conductance are zero for bias voltages below the threshold  $\omega_J = \omega_{\text{res}}$  and diverge exactly at the resonance. At finite temperatures the divergence is reduced: the current reduces to a peak, the conductance reduces to a peak followed by a dip. When  $L_T$ , the length set by temperature, becomes smaller than  $\ell_B$  the resonance becomes fully washed out and disappears. We set  $|\Gamma|^2_{\omega_{\text{res}}} 2^{2(g_c+g_n)-2} e e^* \equiv 1$  and  $W=10\ell_B$ .

neling quasiparticle. The origin for the resonance has a simple explanation. The interference of a tunneling quasiparticle between two paths, separated by a distance  $x$  along the edge, is guided by two phases: on the one hand there is the Aharonov-Bohm phase  $(e^*/e)xW/\ell_B^2$  that basically multiplies the quasiparticle charge  $e^*$  with the flux enclosed in the area  $Wx$ ; on the other hand there is the phase  $\omega_J t$  that is introduced by an applied bias voltage  $V$  between the two edges, with Josephson frequency  $\omega_J = e^*V/\hbar$  and  $t=x/v$ . The resonance occurs at the stated voltage  $V_{\text{res}}$  when the two phases become equal and give rise to constructive interference. The resonance condition follows from the interference among multiple tunneling paths along the length  $L$  of the QLC; however, notice that the length of the channel drops out of the resonance condition Eq. (1). The resonance becomes sharper for longer lengths  $L$  of the QLC. At finite temperature  $T$  the resonance will be reduced and for temperature  $2\pi T > e^*V_{\text{res}}$  it will be washed out. A sharp resonance in the tunneling current will lead to a strong peak followed by a strong dip in the tunneling conductance at nonzero bias.

Now, if there are multiple edge velocities associated with propagation of the quasiparticle along the edge, there are, in principle, multiple phases  $\omega_J x/v_i$ , one for each velocity. We are especially interested in a situation where there are two velocities: one fast velocity associated with the charged mode and one slow velocity associated with the neutral mode(s). For the charged mode the edge velocity is expected on general grounds to be determined by the scale set by electron-electron interactions,  $v_c \sim (e^2/\epsilon)/\hbar = \alpha c/\epsilon$ , where  $\epsilon$  is the dielectric constant of the medium ( $\epsilon_{\text{GaAs}} \approx 12.9$ ). Therefore, the charge-mode velocity is of order  $\sim 10^5$  m/s.

With a width  $W \sim 10\ell_B$  and  $\ell_B \sim 10$  nm we would find  $V_{\text{res}} \sim 0.1$  V  $\approx 10^3$  K; the current that would have to be driven through the sample at such a voltage would surely destroy the quantum Hall state. A resonance due to such a fast velocity is thus not likely experimentally accessible at a QLC. A neutral mode velocity is not bound to the scale set by Coulomb interactions though and can, in principle, be orders of magnitude smaller.

We proceed in Sec. II with a detailed calculation of the tunneling current to determine the precise line shape of the resonance; Fig. 2 illustrates the main result of this paper. In Sec. III we focus on the range of accessible slow edge velocities and compare the observation ranges of the QLC and the 2PC. We conclude in Sec. IV.

## II. TUNNELING CURRENT THROUGH A QUANTUM LONG CONTACT

In this section we calculate the tunneling current through a QLC to determine the line shape of the resonance as a function of bias, temperature, tunneling exponent, and edge velocity. The tunneling current due to  $N$  (discrete) tunneling sites was calculated in linear response in Ref. 16,

$$I_{\text{tun}}(\omega_J) = e^* \sum_{i,j=1}^N \frac{\Gamma_i \Gamma_j^* + \Gamma_i^* \Gamma_j}{2} \int_{-\infty}^{\infty} dt e^{i\omega_J t} \times P_{g/2}(t + x_{ij}/v) P_{g/2}(t - x_{ij}/v) - (\omega_J \leftrightarrow -\omega_J). \quad (2)$$

Here  $x_{ij} = x_i - x_j$  and edge quasiparticle propagator  $P_{g/2}(t)$  is given by

$$P_{g/2}(t) = \begin{cases} \frac{1}{(\delta + it)^g} & \text{for } T = 0, \delta = 0^+, \\ \frac{(\pi T)^g}{(\delta + i \sinh \pi T)^g} & \text{for } T \neq 0. \end{cases} \quad (3)$$

In this paper we will generalize Eq. (2) by making the discrete number of tunneling sites into a continuous distribution,  $\Gamma_i \rightarrow \gamma(x)$ , and to separate contributions from charged and neutral modes, which come with distinct edge velocities  $v_{c/n}$  and tunneling exponents  $g_{c/n}$ ,

$$I_{\text{tun}}(\omega_J) = e^* \int dx dy \gamma(x) \gamma^*(y) \int_{-\infty}^{\infty} dt e^{i\omega_J t} P_{g_c/2} \left( t + \frac{x-y}{v_c} \right) \\ \times P_{g_c/2} \left( t - \frac{x-y}{v_c} \right) P_{g_n/2} \left( t + \frac{x-y}{v_n} \right) P_{g_n/2} \left( t - \frac{x-y}{v_n} \right) \\ - (\omega_J \leftrightarrow -\omega_J). \quad (4)$$

See Fig. 1 for a sketch of the setup. We assume that the entire bulk has the same filling fraction and the edges are the modes associated with that bulk state. In the narrow region under the QLC we do not allow bulk quasiparticles to become trapped.

The form we choose for the tunneling amplitude  $\gamma(x)$  explicitly contains the Aharonov-Bohm phase linear in  $x$ ,

$$\gamma(x) = \frac{\Gamma}{\sqrt{\pi \ell_B}} e^{-x^2/L^2} e^{i(x/L)(e^*/e)N_\Phi}, \quad N_\Phi = \frac{WL}{\ell_B^2}. \quad (5)$$

Here  $N_\Phi$  is  $2\pi$  times the number of flux quanta enclosed in the area  $WL$ ;  $\Gamma/\ell_B$  is a measure of the tunneling amplitude strength per unit length, which is assumed to be small enough to warrant the weak-tunneling approximation of linear response. We included a Gaussian envelope to provide a smooth cutoff scale at length  $L$ ; the Gaussian form simplifies the integration over  $x$  and  $y$ . The exact form of the cutoff is not important when  $L$  is large, and this is the regime we are interested in, because temperature will introduce another, smaller, cutoff length scale. [For the case when  $L$  is not so large (i.e.,  $L/\ell_B \sim 1$ ), the approximation to  $\gamma(x)$  in Eq. (5) is less accurate in that the Aharonov-Bohm phase should not be simply linear but should contain a quadratic piece to account for the funneling in and out of the tunneling region.]

One can carry out the integrals over  $x$  and  $y$  after recasting the expression for the tunneling current Eq. (4) in terms of the (inverse) Fourier transforms of  $P_g(t)$ ,

$$\tilde{P}_g(\omega) = \begin{cases} \theta(\omega) |\omega|^{2g-1} \frac{2\pi}{\Gamma(2g)} & \text{for } T = 0, \\ (2\pi T)^{2g-1} B\left(g + i\frac{\omega}{2\pi T}, g - i\frac{\omega}{2\pi T}\right) e^{\omega/2T} & \text{for } T \neq 0, \end{cases} \quad (6)$$

where  $\theta(s)$  is the Heaviside unit-step function and  $B(a, b)$  is the Euler beta function. In the limit  $v_c \gg v_n$ , i.e., neutral mode much slower than charged mode, the expression for the tunneling current becomes

$$\lim_{v_c \rightarrow \infty} I_{\text{tun}} = e^* |\Gamma|^2 \frac{L^2}{\ell_B^2} \int \frac{d\omega_1}{2\pi} \frac{d\omega_2}{2\pi} \tilde{P}_{g_n/2}(\omega_1) \tilde{P}_{g_n/2}(\omega_2) \\ \times \tilde{P}_{g_c}(\omega_J - \omega_1 - \omega_2) e^{-1/2N_\Phi^2 [1 - (\omega_1 - \omega_2)/\omega_{\text{res}}]^2} \\ - (\omega_J \leftrightarrow -\omega_J), \quad (7)$$

where  $N_\Phi^* \equiv (e^*/e)N_\Phi$ ,  $e^*V_j \equiv \omega_j$ , and  $\omega_{\text{res}} \equiv e^*V_{\text{res}}$  is defined with respect to the neutral velocity as in Eq. (1).

Let us first consider Eq. (7) in the limit of large  $L$ , hence large  $N_\Phi^*$ , in which case the Gaussian in Eq. (7) reduces to a delta function that sets  $\omega_1 - \omega_2 = \omega_{\text{res}}$  (and a prefactor  $\sqrt{2\pi} \omega_{\text{res}}/N_\Phi^*$ ); in the limit of zero temperature one obtains

$$I_{\text{tun}} \rightarrow e |\Gamma|^2 \frac{L}{W} \text{sgn}(\omega_J) \theta(|\omega_J| - \omega_{\text{res}}) \omega_{\text{res}}^{2(g_c+g_n)-1} \\ \times \frac{2^{-g_n} (2\pi)^{3/2}}{\Gamma(g_n) \Gamma(g_n + 2g_c)} \left( \frac{|\omega_J|}{\omega_{\text{res}}} - 1 \right)^{2g_c+g_n-1} \\ \times F\left(1 - g_n, g_n; 2g_c + g_n; \frac{1}{2} - \frac{1}{2} \frac{|\omega_J|}{\omega_{\text{res}}}\right), \quad (8)$$

where  $F$  is the hypergeometric function. Notice the step function  $\theta(|\omega_J| - \omega_{\text{res}})$  so that, at  $T=0$ , the current vanishes for biases below a threshold set by the resonance. Near the resonance, the current scales as  $I_{\text{tun}} \sim (|\omega_J|/\omega_{\text{res}} - 1)^{2g_c+g_n-1}$ . At large biases, far from the resonance, the current scales as

$$I_{\text{tun}} \sim (|\omega_J|/\omega_{\text{res}})^{2g_c-1} \begin{cases} 1 & g_n < \frac{1}{2} \\ \ln|\omega_J|/\omega_{\text{res}} & g_n = \frac{1}{2} \\ (|\omega_J|/\omega_{\text{res}})^{2g_n-1} & g_n > \frac{1}{2} \end{cases}. \quad (9)$$

Next, we consider Eq. (7) for finite length  $L$  and nonzero temperature  $T$ . We find that either will smoothen the divergence at the resonance that exists for  $T=0$  and  $L \rightarrow \infty$ . Note that the ratio  $I_{\text{tun}}/L$  is a useful quantity to compare different lengths  $L$ . The effect of finite temperature is remarkably similar to that of finite length in the sense that we can define a length scale  $L_T$  set by temperature such that

$$\lim_{L \rightarrow \infty} \frac{1}{L} I_{\text{tun}}(L, T \neq 0) \simeq \frac{1}{L_T} I_{\text{tun}}(L_T, T = 0), \quad (10)$$

$$\frac{L_T}{\ell_B} \equiv \frac{e^*V_{\text{res}}}{2\pi T}, \quad L_T = \frac{v_n}{2\pi T} \frac{e^*W}{e\ell_B}. \quad (11)$$

It was already emphasized by Bishara and Nayak<sup>17</sup> for a two-point-contact interferometer that  $v_n/T$  sets a temperature decoherence length scale; they define a temperature decoherence length as  $L_\phi = v_n/(2\pi T g_n)$  (for  $v_c \rightarrow \infty$ ). Their definition differs from ours by a factor of order 1 [since the two setups are different, exact comparison is not possible].

Plots of the tunneling current  $I_{\text{tun}}$  and the differential tunneling conductance  $G_{\text{tun}} = dI_{\text{tun}}/dV$  are shown in Fig. 2 for the following three quantum Hall states: the  $\nu = \frac{5}{2}$  Pfaffian state ( $e^* = e/4$ ,  $g_c = 1/8$ , and  $g_n = 1/8$ ), the  $\nu = \frac{5}{2}$  anti-Pfaffian state ( $e^* = e/4$ ,  $g_c = 1/8$ , and  $g_n = 3/8$ ), and the Abelian  $\nu = \frac{2}{3}$  state ( $e^* = e/3$ ,  $g_c = 1/6$ , and  $g_n = 1/2$ ). The current and conductance are plotted as a function of bias voltage and at different temperatures as indicated by  $L_T$ . The tunneling cur-

rent for a QLC is the main result of this paper, we plot the differential tunneling conductance as well because it is the conductance which is usually measured in experiment.

Qualitatively the resonance at a QLC is independent of tunneling exponents  $g_c$  and  $g_n$ , as the plots for the three different states in Fig. 2 show more or less the same behavior: at zero temperature the current and conductance are strictly zero below the resonance and diverge exactly at the resonance bias voltage of the QLC; at finite temperatures the resonance shows up as a strong peak in the current around the resonance bias voltage (strong peak followed by dip in the conductance) which becomes washed out if temperature becomes too high. Note that  $I_{\text{tun}}(V)$  decays as power law for  $V \gg T$  so  $G_{\text{tun}}$  will be negative here. Qualitatively the resonance is a probe of a slow edge velocity. Quantitatively, the tunneling exponents  $g_c$  and  $g_n$  do affect the detailed shape of the resonance peak at finite temperature and a precise observation of a resonance not only conveys information about the slow edge velocity but also about the tunneling exponents  $g_c$  and  $g_n$  (Ref. 18).

### III. ACCESSIBLE EDGE VELOCITIES

We would now like to address which range of slow edge velocities can realistically be observed and directly compare with the two-point-contact interferometer setup.<sup>16,17</sup> The lower bound is set by temperature (for both setups). For the QLC, the scale  $L_T/\ell_B \approx 1$  is the crossover region where the resonance disappears. The lower bound  $v_{\text{min}}^{\text{QLC}}$  on the slow edge velocity is then given by

$$v_{\text{min}}^{\text{QLC}} \approx \frac{2\pi}{\left(\frac{e^*}{e}\right)\left(\frac{W}{\ell_B}\right)} \frac{k_B T}{\hbar} \ell_B. \quad (12)$$

For typical values,  $T_{\text{base}} = 10$  mK,  $\ell_B = 10$  nm,  $W/\ell_B = 10$ , and  $e^* = e/3$ , we find  $v_{\text{min}}^{\text{QLC}} \approx 25$  m/s. For the 2PC setup, the interference signal (which carries the edge-velocity signature) is washed out when the spacing  $x$  between the two contacts, i.e., the interferometer arm length, is smaller than  $L_\phi$ . In current experiments, device fabrication limits  $x \gtrsim 1$   $\mu\text{m}$ . With  $g_n = 1/4$ , this gives a lower bound of  $v_{\text{min}}^{\text{2PC}} \approx 2000$  m/s. Note that the QLC is sensitive to edge velocities up to two orders of magnitude slower compared to the 2PC setup. An intuitive explanation for this difference is to think of the QLC as an array of point contacts with a very small effective spacing  $x$  which is much smaller than any spacing  $x$  that can be fabricated for a 2PC setup.

For both the QLC and 2PC setups, the upper bound on the edge velocity that can be observed is given by the maximum voltage that can be applied to the quantum Hall system without destroying it due to, e.g., heating (a current  $I = V/R_H$  has

to flow through the system). This maximum voltage  $V_{\text{max}}$  is not as clear cut and may depend on sample, specific experimental setup, and filling fraction. In terms of this  $V_{\text{max}}$  we have for the QLC setup

$$v_{\text{max}}^{\text{QLC}} = \frac{1}{\left(\frac{W}{\ell_B}\right)} \frac{eV_{\text{max}}}{\hbar} \ell_B. \quad (13)$$

To give a numerical estimate, for  $eV_{\text{max}} = 750k_B T_{\text{base}}$  one would find  $v_{\text{max}}^{\text{QLC}} = 1000$  m/s. The bulk excitation gap  $T_{\text{gap}}$  likely sets the scale for  $V_{\text{max}}$  but prefactors are important [e.g.,  $eV_{\text{max}} \approx T_{\text{gap}}$  and  $e^*V_{\text{max}} \approx 2\pi T_{\text{gap}}$  differ by a factor 20]. For the 2PC setup our estimate gives  $v_{\text{max}}^{\text{2PC}} \approx 10^5$  m/s (for  $x = 1$   $\mu\text{m}$ ).

### IV. CONCLUSION

Given our estimates of the (nonoverlapping) ranges of accessible edge velocities, we have to conclude that the QLC and 2PC setups complement each other quite well. A dedicated search for slow edge velocities should implement both setups in order to probe edge velocities from tens to ten thousands of meters per second. Besides the different ranges of edge velocities, the main difference between the two setups is the signature of the edge velocity: for the QLC it is a *resonance* in the tunneling conductance as function of bias, for the 2PC it is a *modulation* of the interference signal within the tunneling conductance as function of bias,<sup>16,17</sup> detecting a modulation in interference requires an extra experimental knob compared to detecting a resonance.

In this paper we assume the width  $W$  of the QLC is constant but disorder may lead to fluctuations of the width. As long as such fluctuations along the edge occur on scales larger than the magnetic length the resonance should survive, albeit with some broadening of the line shape. A feature at finite bias observed in device 2 of Ref. 19, a channellike geometry, can be due to a resonance, and leads us to expect that the proposed QLC setup is physically realizable.

In summary, we proposed and analyzed a device that can potentially detect the presence of neutral edge modes at the edge of QH liquids, by resolving velocities as small as tens of m/s. The ability to resolve these modes and measure their velocity of propagation using a QLC (possibly combined with a 2PC) can provide a better quantitative understanding of QH edge states and can help guide attempts to probe quasiparticle statistics, both Abelian and non-Abelian.

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<sup>1</sup>B. I. Halperin, Phys. Rev. B **25**, 2185 (1982).

<sup>2</sup>X.-G. Wen, Int. J. Mod. Phys. B **6**, 1711 (1992).

<sup>3</sup>C. de C. Chamon and X. G. Wen, Phys. Rev. B **49**, 8227 (1994).

<sup>4</sup>A. H. MacDonald, Phys. Rev. Lett. **64**, 220 (1990).

<sup>5</sup>C. L. Kane, M. P. A. Fisher, and J. Polchinski, Phys. Rev. Lett.

**72**, 4129 (1994).

<sup>6</sup>G. Moore and N. Read, Nucl. Phys. B **360**, 362 (1991).

<sup>7</sup>M. Levin, B. I. Halperin, and B. Rosenow, Phys. Rev. Lett. **99**, 236806 (2007); S.-S. Lee, S. Ryu, C. Nayak, and M. P. A. Fisher, *ibid.* **99**, 236807 (2007).

- <sup>8</sup>B. J. Overbosch and X.-G. Wen, arXiv:0804.2087 (unpublished).
- <sup>9</sup>R. C. Ashoori, H. L. Stormer, L. N. Pfeiffer, K. W. Baldwin, and K. West, Phys. Rev. B **45**, 3894 (1992).
- <sup>10</sup>N. B. Zhitenev, R. J. Haug, K. v. Klitzing, and K. Eberl, Phys. Rev. Lett. **71**, 2292 (1993).
- <sup>11</sup>A. M. Chang, L. N. Pfeiffer, and K. W. West, Phys. Rev. Lett. **77**, 2538 (1996).
- <sup>12</sup>M. Grayson, D. C. Tsui, L. N. Pfeiffer, K. W. West, and A. M. Chang, Phys. Rev. Lett. **80**, 1062 (1998).
- <sup>13</sup>A. V. Shytov, L. S. Levitov, and B. I. Halperin, Phys. Rev. Lett. **80**, 141 (1998).
- <sup>14</sup>D.-H. Lee and X.-G. Wen, arXiv:cond-mat/9809160 (unpublished).
- <sup>15</sup>A. Lopez and E. Fradkin, Phys. Rev. B **59**, 15323 (1999).
- <sup>16</sup>C. de C. Chamon, D. E. Freed, S. A. Kivelson, S. L. Sondhi, and X. G. Wen, Phys. Rev. B **55**, 2331 (1997).
- <sup>17</sup>W. Bishara and C. Nayak, Phys. Rev. B **77**, 165302 (2008).
- <sup>18</sup>The quasiparticle *charge*  $e^*$  can be probed as well; however this requires a measurement of the tunneling current *noise* in addition to a measurement of tunneling current (conductance).
- <sup>19</sup>I. P. Radu, J. B. Miller, C. M. Marcus, M. A. Kastner, L. N. Pfeiffer, and K. W. West, Science **320**, 899 (2008).